Benha University
Faculty Of Engineering at Shoubra

ECE 411
Antennas & Wave propagations
(2016/2017)
Lecture (3)
Antenna Parameters

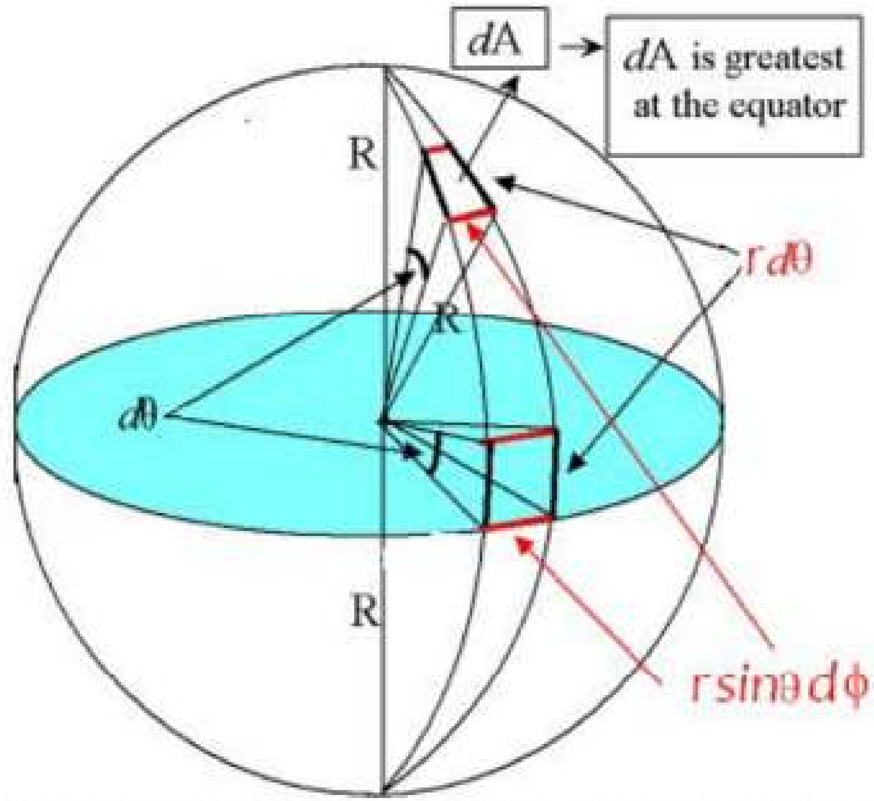
Prepared By :
Dr. Moataz Elsherbini
motaz.ali@feng.bu.edu.eg

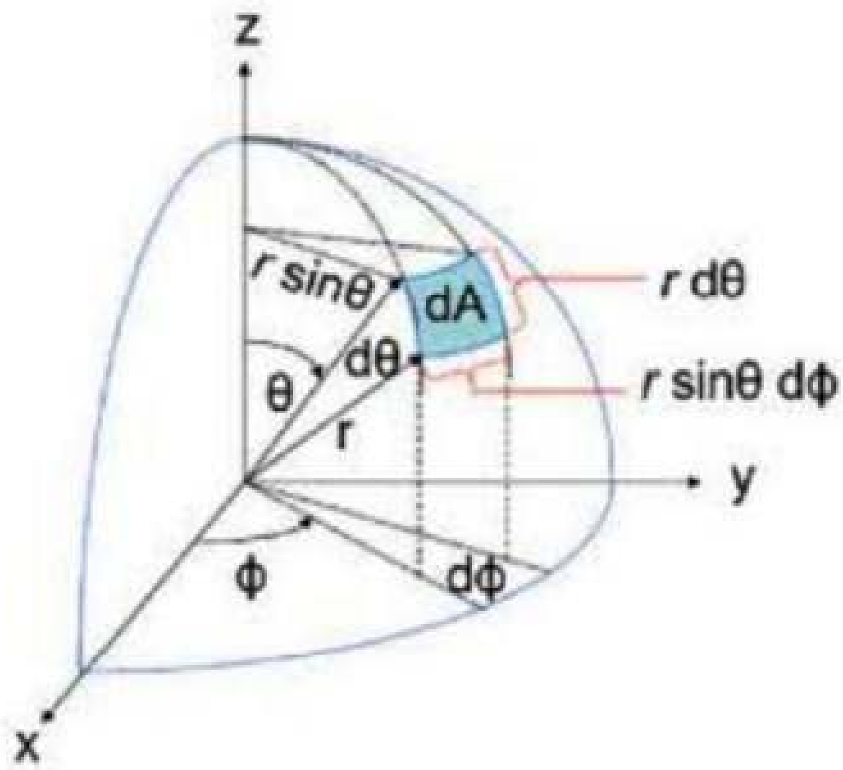
Agenda

- Remember (Solid angle & Directivity)
- Power density
- Radiation intensity and Radiated power
- Beam efficiency
- Gain
- Radiation Resistance
- Effective and Physical Aperture
- Frris Transmission equation

1 - Remember (Solid angle & Directivity)

Remember (Solid angle & Directivity)





Remember (Solid angle & Directivity)

$$D = \frac{4\pi}{\Omega_A}$$

Exact

$$\Omega_A = \iint_{4\pi} P_n(\theta,\varphi)\,d\Omega = \iint_{4\pi} U_n(\theta,\varphi)\,d\Omega = \iint_{4\pi} E_n^2(\theta,\varphi)\,d\Omega$$

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

Approximate (Sr)

$$\Omega_A = \Theta_{HP}(rad) * \Phi_{HP}(rad)$$

Approximate (degree square)

$$D = \frac{41253}{\Theta^o_{HP} * \Phi^o_{HP}}$$

2 - Power density

Power density

Average Poynting vector:

$$\mathbf{P}_{\mathrm{av}} = \frac{1}{2}\mathrm{Re}\left\{\mathbf{E}(x,y,z)\times\mathbf{H}(x,y,z)^{*}\right\} \quad (\mathrm{W/m^2})$$

Note that Poynting vector is a *real* vector. Its magnitude gives the instantaneous or average **power density** of the electromagnetic wave. Its direction gives the direction of the power flow at that particular point.

$$W_{av} = \frac{1}{2}\mathrm{Re}(E\,\chi H^{*}) = \frac{U}{r^2}(W\ /\ m^2)$$

$$P_{rad} = \oiint_{4\pi} W_{av}\ dA = \oiint_{4\pi} \frac{1}{2}\mathrm{Re}(E\,\chi H^{*}).\,(r^2 Sin\,\theta d\,\theta d\,\phi)$$

Example (1)

A hypothetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field (E_θ) is measured to be 5 V/m. Find the

(a) Power density (W_{rad})

(b) Power radiated (P_{rad})

(a) $\underline{W}_{rad} = \frac{1}{2}[\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta}\hat{a}_r = \frac{5^2 \hat{a}_r}{2(120\pi)} = 0.03315\ \hat{a}_r$ watts/m²

(b) $P_{rad} = \oint_S W_{rad}\, dS = \int_0^{2\pi}\int_0^{\pi} (0.03315)(r^2 \sin\theta\, d\theta\, d\phi)$

$= \int_0^{2\pi}\int_0^{\pi} (0.03315)(100)^2 \sin\theta\, d\theta\, d\phi$

$= 2\pi\,(0.03315)(100)^2 \cdot \int_0^{\pi} \sin\theta\, d\theta = 2\pi(0.03315)(100)^2 \cdot 2$

$= 4165.75$ watts

3- Radiation intensity and Radiated power

Radiation intensity and Radiated power

Radiation intensity in a given direction is the power per unit solid angle radiated in this direction by the antenna.

$$U = \frac{dP_{rad}}{d\Omega} \; W/sr \quad \Rightarrow \quad P_{rad} = \oiint_{4\pi} U \, d\Omega$$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} U_n(\theta,\varphi) d\Omega} = \frac{4\pi U_{\max}}{\iint_{4\pi} U(\theta,\varphi) d\Omega} = \frac{4\pi U_{\max}}{P_{rad}}$$

$$U = r^2 * W_{av}$$

Example (2)

The radiation intensity of antenna is given by $U=B_o Cos\theta$. U exists only in the upper hemisphere, Find

The exact directivity , The approximate directivity , The decibel difference.

$U = U_n = P_n = Cos\theta.$

$$(a)\ Dexact = \frac{4\Pi}{\int_0^{2\Pi}\int_0^{\frac{\Pi}{2}} Cos\,\theta\ Sin\theta d\theta d\phi} = \frac{4\Pi}{(2\Pi)(\frac{-Cos^2\theta}{2})_0^{\frac{\Pi}{2}}} = 4.$$

$$(b) D\ approximate = \frac{4\Pi}{\theta_{HP}\phi_{HP}}. = \frac{4\Pi}{\theta_{HP}\phi_{HP}} = \frac{4\Pi}{(\theta_{HP})^2}|_{sr} = \frac{41253}{(\theta_{HP})^2}|_{deg2}.$$

We calculate θ_{max} ➔ $(Cos\theta_{max} = 1)$ ➔ at $\theta_{max} = 0^o$,

We calculate θ_h ➔ $(Cos\theta_h = \frac{1}{2})$ ➔ $\theta_h = 60^o$

$\theta_{HP} = 2*|\theta_{max} - \theta_h| = 2*|0^o - 60^o| = 120^o$

so : $Dapprox. = \frac{41253}{(\theta_{HP})^2}|_{deg2}. = = \frac{41253}{(120)^2} = 2.86.$

(c) $Decibel\ difference = 10\log\frac{4}{2.86} = 1.46db.$

Example (3)

The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of

$U=B_o\cos^3\theta$ (watts/unit solid angle) ($0\le\theta\le\pi/2$, $0\le\phi\le2\pi$)

Find the

(a) Maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.

(b) Exact Directivity of the antenna (dimensionless and in dB).

(c) Exact Gain of the antenna (dimensionless and in dB).

$$U = B_o \cos^3\theta$$

(a) $$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \sin\theta \, d\theta \, d\phi = B_o \int_0^{2\pi}\int_0^{\pi/2} \cos^3\theta \sin\theta \, d\theta \, d\phi$$

$$= 2\pi B_o \int_0^{\pi/2} \cos^3\theta \sin\theta \, d\theta$$

$$P_{rad} = 2\pi B_o \left(-\frac{\cos^4\theta}{4}\right)\Big|_0^{\pi/2} = \frac{\pi}{2} B_o = 10 \Rightarrow B_o = \frac{20}{\pi} = 6.3662$$

$$U = 6.3662 \cos^3\theta$$

$$W = \frac{U}{r^2} = \frac{6.3662}{r^2}\cos^3\theta = \frac{6.3662}{(10^3)^2}\cdot\cos^3\theta = 6.3662\times10^{-6}\cos^3\theta$$

$$W|_{max} = 6.3662\times10^{-6}\cdot\cos^3\theta\Big|_{max} = 6.3662\times10^{-6} \text{ watts/m}^2$$

(b) $$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(6.3662)}{10} = 8 = 9\,\text{dB}$$

(c) $$G_o = e_t D_o = 8 = 9\,\text{dB}$$

4- Beam efficiency

Beam efficiency

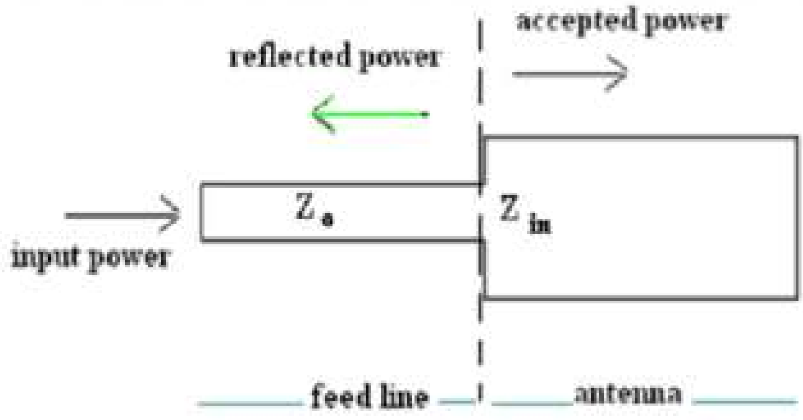
$$e_t = e_{cd} * e_r$$

Where: $e_t = P_{rad}/P_{input}$ total efficiency

$e_{cd} = P_{rad}/P_{accept}$ radiation efficiency

Contains conduction and dielectric losses

$e_r = 1 - |\Gamma|^2$ Mismatch loss



$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0},$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}.$$

Γ = voltage reflection coefficient at the input terminals of the antenna Z_{in} = antenna input impedance, Z_0 = characteristic impedance of the transmission line. VSWR = voltage standing wave ratio

$$e_{cd} = \frac{R_r}{R_L + R_r} \quad \text{(dimensionless)}$$

5- Gain

Gain

Gain

- Defined as ratio of radiation intensity in a given direction to radiation intensity obtained if accepted power where radiated isotropic.
- Gain does not account for losses arising from impedance mismatches

$$G(\theta,\phi) = e_{cd}\left[4\pi \frac{U(\theta,\phi)}{P_{rad}}\right]$$

$$G(\theta,\phi) = e_{cd} D(\theta,\phi)$$

$$e_{cd} = \frac{R_r}{R_L + R_r} \quad \text{(dimensionless)}$$

the maximum value of the gain

$$G_0 = G(\theta,\phi)|_{max} = e_{cd} D(\theta,\phi)|_{max} = e_{cd} D_0$$

Absolute Gain

Take into account losses arising from impedance mismatches

$$G_{abs} = e_r\, G(\theta,\phi) = (1-|\Gamma|^2) G(\theta,\phi) = e_r e_{cd} D(\theta,\phi) = e_t D(\theta,\phi).$$

where

$e_r = (1-|\Gamma|^2)$, reflection (mismatch) efficiency,

e_t = total efficiency

- For the maximum values

$$G_{0abs} = e_t D_0.$$

$$P_{rad} = e_t * P_{in}$$

Example (4)

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by $U=B_o\sin^3\theta$. Find the maximum gain and maximum absolute gain of this antenna.

$$U|_{\max} = U_{\max} = B_0$$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta \, d\phi = 2\pi B_0 \int_0^{\pi} \sin^4\theta \, d\theta = B_0\left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi \frac{U_{\max}}{P_{\text{rad}}} = \frac{16}{3\pi} = 1.697$$

Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$.

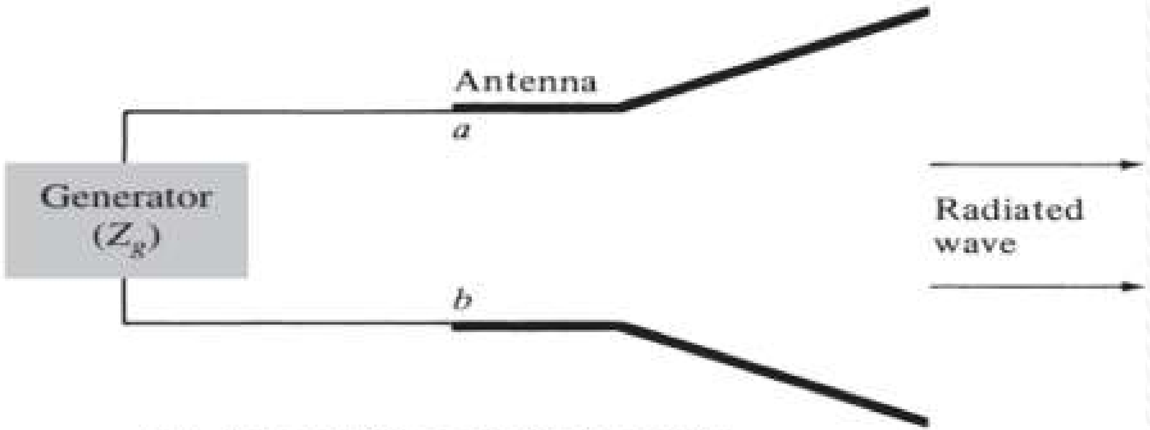
$$G_0 = e_{cd} D_0 = 1(1.697) = 1.697$$

$$e_r = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73-50}{73+50}\right|^2\right) = 0.965$$

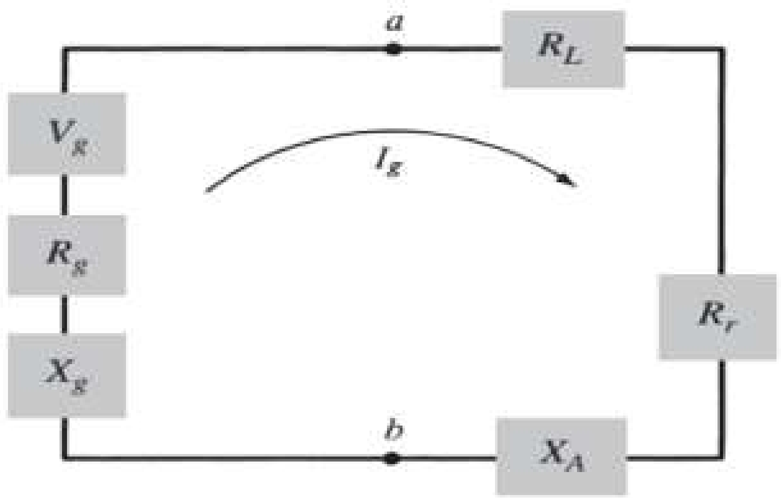
$$G_{0abs} = e_0 D_0 = 0.965(1.697) = 1.6376$$

6- Radiation Resistance

Radiation Resistance



(a) Antenna in transmitting mode



(b) Thevenin equivalent

Radiation Resistance

$$R_{rad} = \frac{2P_{rad}^{total}}{|I_o|^2} = \frac{2\oiint\limits_{4\pi} U(\theta,\varphi)\,d\Omega}{|I_o|^2}$$

For Infinitesimal Dipole

$$R_{rad} = 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2$$

For Short Dipole

$$R_{rad} = 20\pi^2 \left(\frac{\ell}{\lambda}\right)^2$$

Example (5)

Find R_r of a unidirectional pattern of antenna with $U=8Sin^2\theta\, Sin^3\phi$ wsr^{-1}, where $0\leq \theta \leq \Pi$ & $0\leq \phi \leq \Pi$. If I_{rms}=3A.

$$P_{rad}=\int_0^{\Pi}\int_0^{\Pi} U d\Omega = \int_0^{\Pi}\int_0^{\Pi}(8Sin^2\theta Sin^3\phi) * Sin\theta d\theta d\phi = I^2 R_r$$

$$R_r = 1.6\ \Omega$$

Example (6)

An isotropic antenna has a field pattern given by $E=10\ I_o /r$ V/m, where I is the amplitude of current, r is distance (m), find R_r. repeat for hemisphere antenna.

$$P= SA = \frac{E^2}{Z}A = 0.5\frac{100I^2}{r^2 * Z}(4\Pi * r^2) \Rightarrow \text{for hemisphere } A=(2\Pi * r^2)$$

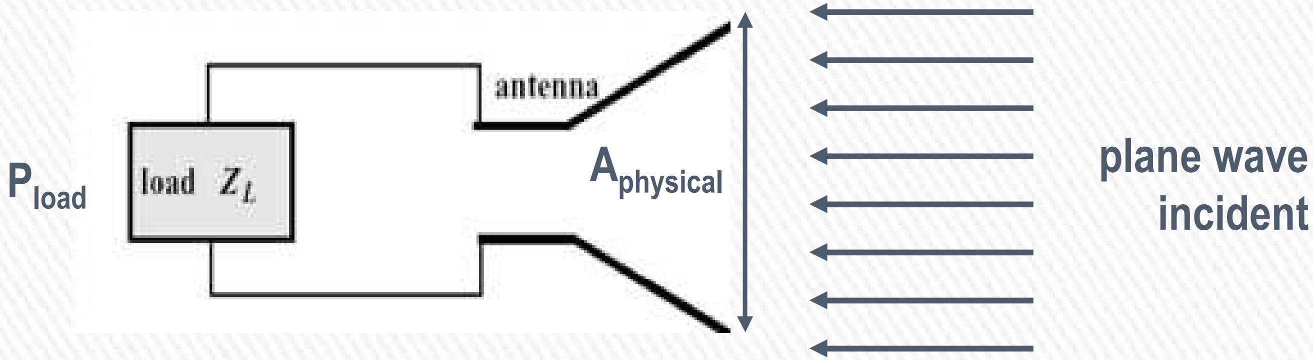
$$P= 0.5I_o{}^2R_r$$

$$So\ 0.5\frac{100I_o{}^2}{r^2 * Z}(4\Pi * r^2) = 0.5I_o{}^2R_r \rightarrow R_r= 3.33\Omega$$

7- Effective and Physical Aperture

Effective and Physical Aperture

The ***effective antenna aperture*** is the ratio of the available power at the terminals of the antenna to the power flux density of a plane wave incident upon the antenna, which is polarization matched to the antenna. If there is no specific direction chosen, the direction of maximum radiation intensity is implied.



$$D = \frac{4\pi}{\Omega_A} = 4\pi \frac{A_e}{\lambda^2}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o$$

$$A_e = \xi_{ap} A_{em}$$

Aperture efficiency

$$0 \leq \varepsilon_{ap} \leq 1$$

Example (7)

If the aperture efficiency of an antenna is 0.7 and the beam traveling at 6 GHZ. Calculate the directivity, HPBW, and FNBW (approximately). Given circular aperture of diameter 3 meter.

$$D = \frac{4\Pi}{\lambda^2} * \eta * A_{em} = \frac{4\Pi}{\lambda^2} * 0.7 * (\Pi * r^2) = \frac{4\Pi}{(\frac{3*10^8}{6*10^9})^2} * 0.7 * (\Pi * (1.5)^2) = 24871$$

So D=24871.

$$D = \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{(\theta_{HP})^2} = 24871$$

So $(\theta_{HP}) = 1.28^o$.

$FNBW = 2*(\theta_{HP}) = 2.57^o$.

Example (8)

What is the maximum effective aperture (approximately) for a beam antenna having HPBW of 30º & 35º in perpendicular planes intersecting in the beam axis? Minor lobes are small and may be neglected.

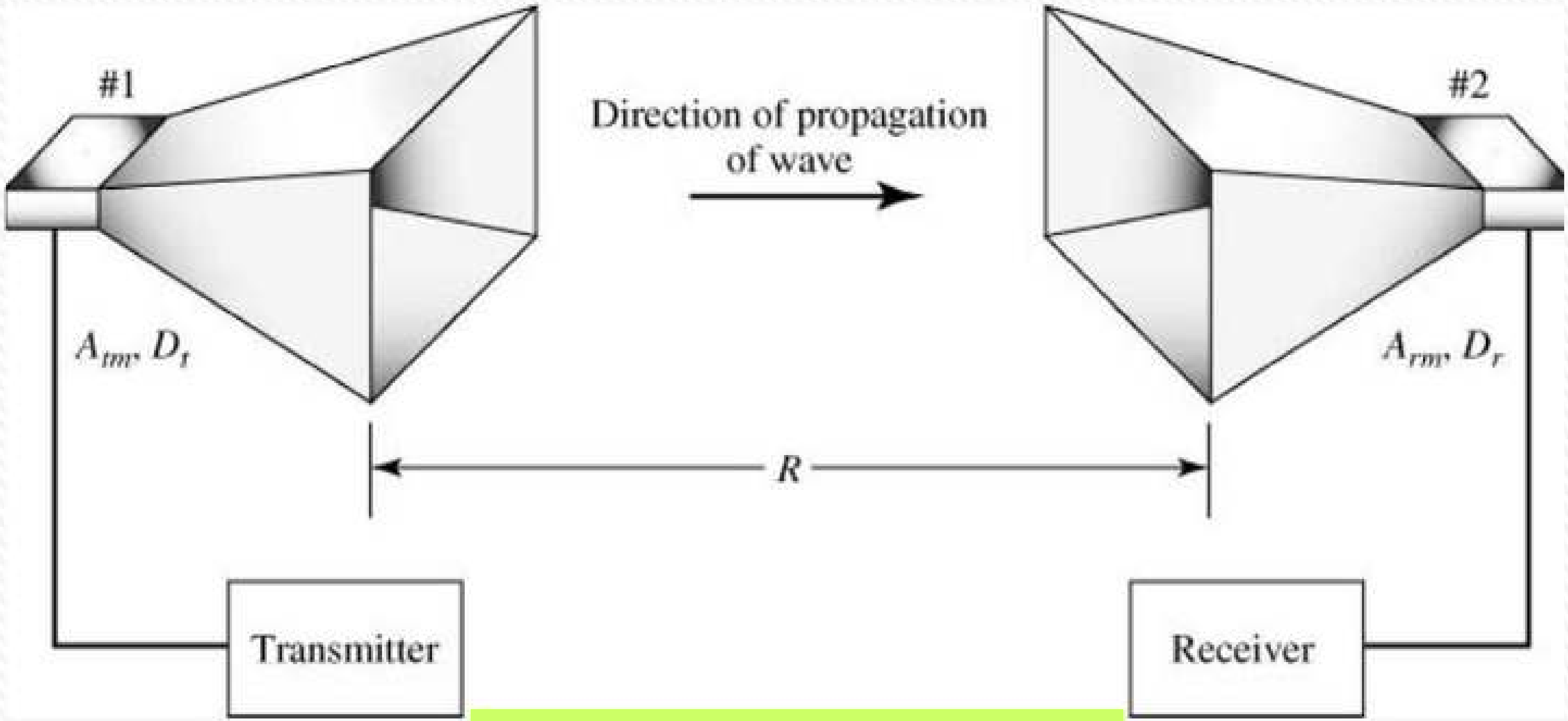
$$D = \frac{4\Pi}{\lambda^2} A_{em} \rightarrow A_{em} = \frac{D}{4\Pi}\lambda^2$$

$$D = \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{30*35} = 39.3$$

$$A_{em} = \frac{D}{4\Pi}\lambda^2 = 3.2\,\lambda^2$$

8- Frris Transmission Equation

Frris Transmission Equation



$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_t G_r$$

Example (9)

What is the max? Power received at a distance of 0.5 Km. over a free-space 1GHZ circuit consisting of a transmitting antenna with 25dB gain and receiving antenna with 20dB gain? The gain is with respect to a lossless isotropic source. The transmitting antenna input is 150W.

$$\lambda = \frac{C}{F} = 3*10^8/1*10^9 = 0.3m.$$

$$\frac{P_r}{P_{in}} = G_{in} G_r \left(\frac{\lambda}{4\Pi R}\right)^2.$$

$$G_r|_{db} = 10 \log G_r \Rightarrow G_r = 100$$

$$G_{in}|_{db} = 10 \log G_{in} \Rightarrow G_{in} = 316.22$$

$$\frac{P_r}{150} = 100 * 316.22 \left(\frac{0.3}{4\Pi * 0.5 * 10^3}\right)^2 \Rightarrow P_r = 10.8mw.$$

Zatoona

$$P_{rad} = \oiint_{4\pi} W_{av}\, dA = \oiint_{4\pi} \frac{1}{2} \mathrm{Re}(E \chi H^*).(r^2 Sin\theta d\theta d\phi) = \oiint_{4\pi} U\, d\Omega = \frac{1}{2} I_o^2 R_r$$

$$P_{rad} = e_t * P_{in}$$

$$U = r^2 * W_{av}$$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} U_n(\theta, \varphi) d\Omega} = \frac{4\pi U_{\max}}{\iint_{4\pi} U(\theta, \varphi) d\Omega} = \frac{4\pi U_{\max}}{P_{rad}}$$

$$e_t = e_r e_{cd}$$

$$e_{cd} = \frac{R_r}{R_L + R_r} \quad \text{(dimensionless)}$$

$$e_r = 1 - |\Gamma|^2$$

$$\Gamma = \frac{Z_{input} - Z_{generator}}{Z_{input} + Z_{generator}}$$

$$G = e_t \cdot D = e_r e_{cd} \cdot D$$

$$e_t \leq 1$$

$$R_{rad} = \frac{2P_{rad}^{total}}{|I_o|^2} = \frac{\frac{2\oiint U(\theta,\varphi)d\Omega}{4\pi}}{|I_o|^2}$$

For Infinitesimal Dipole

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$$A_{em} = \frac{\lambda^2}{4\pi} D_o$$

$$A_e = \xi_{ap} A_{em}$$

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_t G_r$$

Next Lecture

Ch(2): Antenna parameters (Cont.)

Polarization

Ch (3) : Point Sources

Dr. Moataz Elsherbini

motaz.ali@feng.bu.edu.eg


Thank You